NOTES 9-6

Sec 1 H | **Inverse Matrices** | **Unit 9**

Identity Matrix: SQUARE Matrix

- Matrix with 1's on the diagonal & 0's everywhere else.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

If A is any matrix $n \times n$ & I is the $n \times n$ identity then, $A \cdot I = A$

$I \cdot A = A$

Inverses: inverse of A is A^{-1}

- Two matrices are INVERSES of each other if their product is the IDENTITY.

$A \cdot B = I$ $B \cdot A = I$

The Inverse of a 2x2 Matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\det A = ad - cb$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Ex. 1: Find the inverse of $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ $\det A = 6 - 4 = 2$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\cdot 2 & \frac{1}{2}\cdot -1 \\ \frac{1}{2}\cdot -4 & \frac{1}{2}\cdot 3 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}$$

$A \cdot A^{-1} = I$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} \cdot A = I$

Ex. 2: Solve the following equations.

$$\frac{5x}{5} = \frac{-15}{5}$$

$x = -3$

$$\frac{3}{2}x = 27 \cdot \frac{2}{3}$$

$x = 18$

Matrix Equations:

- Find the inverse of matrix A
- To solve the equation for X, multiply both sides of the equation by A^{-1} on the LEFT!

Ex. 3: Solve the matrix equation $AX = B$ for the 2×2 matrix X.

$$\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} X = \begin{bmatrix} 8 & -5 \\ -6 & 3 \end{bmatrix}$$

$A^{-1} \cdot AX = A^{-1} \cdot B$

$IX = A^{-1} \cdot B$

$X = A^{-1} \cdot B$

$\det A = 4 - 3 = 1$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 8 & -5 \\ -6 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & -2 \\ 0 & -3 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -2 \\ 0 & -3 \end{bmatrix}$$

$3x + 2 = 7$

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Ex. 4: Solve the following matrix equation.

$$\begin{bmatrix} 4 & -3 \\ -5 & 5 \end{bmatrix} X + \begin{bmatrix} -7 & 4 \\ -8 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -13 & 8 \end{bmatrix} - \begin{bmatrix} -7 & 4 \\ -8 & 3 \end{bmatrix}$$

$-\begin{bmatrix} -7 & 4 \\ -8 & 3 \end{bmatrix}$ $\quad -13 - -8$

$\det A = 20 - 15 = 5$

$$\begin{bmatrix} 4 & -3 \\ -5 & 5 \end{bmatrix} X = \begin{bmatrix} 10 & -1 \\ -5 & 5 \end{bmatrix}$$

$\frac{3}{8} \cdot \frac{-8}{1}$

$$A^{-1} = \frac{1}{5} \cdot \begin{bmatrix} 5 & 3 \\ 5 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 3/5 \\ 1 & 4/5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3/5 \\ 1 & 4/5 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ -5 & 5 \end{bmatrix} X = \begin{bmatrix} 1 & 3/5 \\ 1 & 4/5 \end{bmatrix} \begin{bmatrix} 10 & -1 \\ -5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 10+3 & -1+5 \\ 10+4 & -1+4 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 6 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 7 & 4 \\ 6 & 3 \end{bmatrix}$$

How do we tell if two matrices are inverses of each other?

↳ $A \cdot B$ & $B \cdot A = I$ Multiply them together & see if you get the identity.

↳ Find A^{-1} & see if it $= B$.